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the central shower will commence at Greenwich on Saturday at midnight, and continue until Sunday morning, at 3^h A.M.; the circumstances in this case being as favourable for its complete observation as they were on the night of the 13th–14th of November, 1866.

Although it does not appear that the outlying currents of the group are this year favourably situated for observation in Great Britain, yet the observation of a frequency of meteors towards daybreak on the morning of the 13th (Saturday) and of an unusual number of meteors after 11 o'clock P.M. on the night of the 14th (Sunday) would confirm the probable existence of two companion streams of the central meteor-current, and would afford material data for determining the limits of their extent.

The large meteor which was very generally seen in the south of England at about $6^h 50^m$ P.M. on the evening of the 6th instant, was also seen at Hawkhurst, in Kent, and the position of the luminous streak which it left visible, standing, apparently, quite perpendicularly over the point where the meteor disappeared, was noted exactly by the stars. It extended from *Vega Lyræ* to a point two or three degrees above and to the right of the star *μ Herculis*, where the meteor burst into fragments of white blue and orange colours, and left a small cloud of light at that spot, like a nebula, which remained visible for several minutes. The observer particularly noticed the extreme rapidity of the meteor's flight, and its perpendicular direction towards the Earth; but no sound of a report was heard, although he listened attentively for some minutes. The direction of the meteor was nearly due west, and its point of disappearance was about 40° from the horizon.

Note on the Sun's Motion in Space and on the relative Distances of the Fixed Stars of various Magnitudes. By Richard A. Proctor, B.A.

Having recently had occasion to examine Mr. Main's Table of the Proper Motions of 1167 Stars, and the conclusions with reference to the Sun's motion deduced from that table by a method devised by the Astronomer Royal, and carried out at his request by Mr. Dunkin, I have been led to notice certain facts which seem to me to be not without significance.

In the first place, I would call attention to the table drawn up by Mr. Dunkin, in which the sums of the squares of the stars' proper motions are compared with the corresponding sums when the proper motion of each star has been corrected for the Sun's calculated motion in space. Mr. Dunkin comments on the singular smallness of the correction thus introduced. And this view seems abundantly verified by the table, which runs as follows, the divisions 1 to 7 corresponding to Struve's arrangement of the stars in order of magnitude:—

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Sums of Squares of Motion in Parallel.

	Uncorrected.	Corrected.
Division 1	2·0637	1·2123
2	1·8743	1·9292
3	9·2894	9·5607
4	6·4732	5·4608
5	43·4126	42·4236
6	14·8637	14·2750
7	0·7814	0·7215
Sum	78·7583	75·5831

ums of Squares of Motion in N.P.D.

	Uncorrected.	Corrected.
Division 1	6·7231	5·6883
2	0·5351	0·7805
3	4·9739	4·7569
4	6·9390	6·5255
5	39·4335	38·7292
6	4·5671	4·3376
7	0·0951	0·0904
Sum	63·2668	60·9084

Commenting on this result, Sir John Herschel remarks:—"No one need be surprised at this. If the Sun move in space, why not also the stars? And if so, it would be manifestly absurd to expect that any movement could be assigned to the Sun by any system of calculation which would account for more than a very small portion of the totality of the observed displacements."

It had always seemed to me that this conclusion might require to be modified if the question were subjected to mathematical scrutiny; my reason for forming this view being this,—that the largeness of the number of stars operates as much to increase the extent of the correction as to increase the amount of the uncorrected squares, since every star is affected by the Sun's motion in space.

It occurred to me recently that the following simple geometrical proof serves to show that the correction to be looked for is much larger than that which Mr. Dunkin's figures exhibit.

Suppose that in any small region of the sky there are a large number of stars (say n), all at the same distance from the Earth, and travelling in all directions with a velocity exactly equal to that with which the Sun is travelling. Let us suppose that the motion of any one of these stars which is travelling at right angles to the line of sight carries the star over an arc p in a year; and that the region of the sky is so situated that the effect of the

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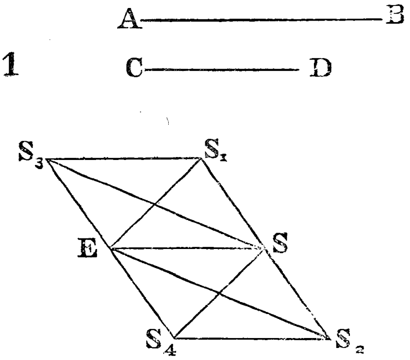
Sun's motion on a star at rest (at the given distance) would be to produce an apparent annual proper motion q'.

Let AB (fig. 1) $= p$
$CD = q'$

Fig1



Suppose two of the stars to move in opposite directions, SS_1, SS_2, along the same straight line, both (we may suppose for convenience) starting from S, and the lines SS_1, SS_2, being in reality equal to AB, but foreshortened. Then the sum of squares for these stars, if unaffected by the Sun's motion, would be

$$= SS_1^2 + SS_2^2,$$

where SS_1, SS_2, represent the *apparent* lengths of these lines.

Suppose that SE is the apparent motion due to the Sun's real motion, so that $SE = CD$.

Complete the parallelograms SS_1S_3E and SES_4S_2, and draw their diagonals. Then the sum of squares for the two stars as affected by the Sun's motion,

$$\begin{aligned} &= SS_3^2 + SS_4^2 \\ &= S_2E^2 + S_1E^2 \\ &= SS_1^2 + SS_2^2 + 2\,ES^2 \end{aligned}$$

Therefore, for every pair of stars moving in opposite directions, there is an increase of $2\,q'^2$ in the sum of squares. Therefore there is an average increase of q'^2 for each star in the region.

Now a moment's consideration will show that, precisely as the lines SS_1 and SS_2 vary from the full length AB to zero, as we vary the supposed motion in all directions around S (for stars in a given small region of the heavens), so does CD vary from AB to zero as we shift the small region over the heavens. It is not merely that the limits of change are the same, but the proportion of lines of a given length SS_1 for the star's motion is exactly the same as the proportion of corrections equal to SS_1 for the Sun's motion. Since, therefore, for each region there is an increase per star equal to the square of the proper motion due to the Sun's motion (for that region), and the several stars of each region have proper motions varying according to exactly the same law as that according to which the effect of the Sun's motion varies over the celestial sphere, it is obvious that for the whole sphere the effect of the Sun's motion must be to increase the sum of squares by an amount exactly equal to the sum of squares due to the stars' own motions. In other words, the sum of squares is doubled

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through the effect of the Sun's motion, if only the Sun be assumed to travel at the same rate through space as the several stars at a given distance from him.

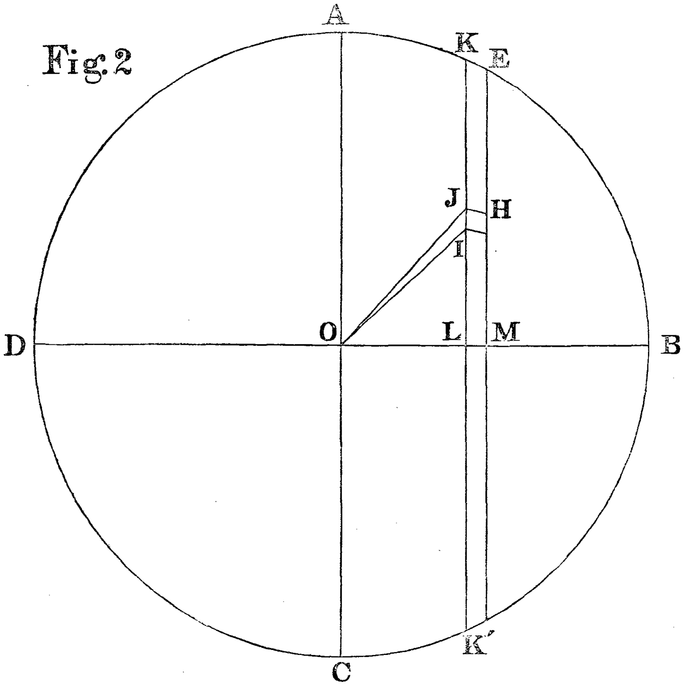
It is obvious also that the same is true if the stars at a given distance move with different velocities, but the Sun's velocity is the mean (its square equal to the mean of sum of squares) of the velocities of the stars at said distance.

Further, if the mean velocity of the stars at a given distance be p, and the Sun's velocity be q, then we have the ratio

$$\frac{\text{Corrected sum of squares}}{\text{Uncorrected sum}} = \frac{p^2}{p^2 + q^2}.$$

Before proceeding to apply this law, I propose to deduce it by another process, which will serve to indicate what is the actual sum of squares, corrected or uncorrected, for a given large number of stars, all at the same distance from the Earth.

Fig. 2



Let A B C D, fig. 2, represent a small circular space on the celestial sphere; and suppose a large number of stars within this space, travelling in all directions with a mean velocity which would give to one moving at right angles to the line of sight an annual proper motion $= p$. Then, in order to determine the mean square of the apparent proper motion — the Earth being supposed to be at rest — we may suppose every star to start from O, O B $= p$, and the points towards which the stars are severally moving, uniformly spread over the sphere A B C D.

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Take now a thin zone of the sphere's surface by parallel planes perpendicular to O B, through L K, M E, where

$$\mathrm{OL} = x, \qquad \mathrm{OM} = x + \delta x.$$

Then the number of stars whose directions lie towards some part of this zone, is (by a well-known property of the sphere),

$$= \frac{n \,.\, \delta x}{2p},$$

n being the total number of stars.

Again, take planes B O I, B O J, inclined at angles θ and $(\theta + \delta\theta)$ to the plane through O B and the observer's eye.

Then the number of stars, whose direction is towards some part of the small element I H of the sphere's surface,

$$= \frac{n\,\delta x}{2p} \cdot \frac{\delta\theta}{2\pi}.$$

When the element I H is taken small enough, each of these stars has an apparent proper motion, whose square

$$= x^2 + (p^2 - x^2)\sin^2\theta.$$

Therefore, it follows that the mean value required

$$= \frac{1}{p\pi}\int_0^{\frac{\pi}{2}}\int_{-p}^{+p}\left\{x^2 + (p^2 - x^2)\sin^2\theta\right\} d\theta\, dx$$

$$= \frac{2p^2}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\cos^2\theta}{3} + \sin^2\theta\right) d\theta$$

$$= \frac{2p^2}{3\pi}\int_0^{\frac{\pi}{2}}\left\{\frac{1 + \cos 2\theta}{2} + \frac{3(1 - \cos 2\theta)}{2}\right\}\delta\theta$$

$$= \frac{2p^2}{3}; \tag{1}$$

and the sum of squares is therefore

$$= \frac{2np^2}{3}. \tag{2}$$

Next let us inquire what the sum would be if the solar system were moving in such a manner that each star within the circle A B C D was affected by a proper motion q (in any direction) due to the Sun's motion.

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Let the motion be parallel to O B, and from right to left. Then it is clear that the only change in the expression to be integrated, is, that for x^2 we must write $(x - q)^2$. Therefore the mean square will be

$$\frac{2p^2}{3} + \frac{1}{p\pi} \int_0^{\frac{\pi}{2}} \int_{-p}^{+p} (q^2 - 2qx)\, d\theta\, dx$$

$$= \frac{2p^2}{3} + q^2. \qquad (3)$$

In this case, then, the sum of squares is

$$= \frac{2np^2}{3} + n q^2. \qquad (4)$$

Lastly, we have to inquire what is the sum of squares for N stars scattered over the whole of the celestial sphere at a given distance R, each affected by the proper motion *p in space*, the Sun being affected by a motion P (p and P representing the arc-motions due to these respective proper motions, when supposed to be taking place in a direction at right angles to the line of sight, and to be seen from a distance R).

In fig. 2, let A B C D now represent the celestial sphere, and take

$$\text{OL} = y$$
$$\text{LM} = \delta y;$$

then the effect of the Sun's motion upon all stars in the zone KMK′ will be to affect them with a proper motion,

$$\text{P} \sin \cos^{-1}\left(\frac{y}{\text{R}}\right)$$

$$= \text{P}\sqrt{\left(1 - \frac{y^2}{\text{R}^2}\right)}.$$

Hence, by (2) the average proper motion of stars in this band

$$= \frac{2p^2}{3} + \text{P}^2\left(1 - \frac{y^2}{\text{R}^2}\right);$$

and, therefore, for the whole sphere, the average proper motion,

$$= \frac{1}{2\text{R}} \int_{-\text{R}}^{+\text{R}} \left\{ \frac{2p^2}{3} + \text{P}^2\left(1 - \frac{y^2}{\text{R}^2}\right) \right\} dy$$

$$= \frac{2p^2}{3} + \text{P}^2 - \frac{\text{P}^2}{3}$$

$$= \frac{2}{3}(p^2 + \text{P}^2). \qquad (5)$$

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and the sum of squares of proper motions

$$= \frac{2\,N}{3}\,(p^2 + P^2) \qquad (6)$$

If $P = p$ (that is, if the Sun's motion be assumed equal to the average motions of the stars) the sum of squares,

$$= \frac{4\,N}{3}\,.\,p^2. \qquad (7)$$

The use of the integral calculus, as above, would only be justified where N is infinite; but where N is considerable, the result must be a close approximation to the truth, with the assumed conditions. And even where N is small, the above result is the most probable, in the case of a random distribution of the N stars, and of the directions of their motion.

Now, if we apply these results to the tables given above, we can determine how far the observed proper motions of the stars are consistent with the supposition that the Sun's proper motion is not very different from the mean proper motion of the stars of different magnitudes; and thence we can form an opinion as to the justice of those estimates of the stars' distances on which the values of the corrections tabulated above have been determined.

First, we require the sums of squares of proper motions, without reference to direction. Since the square of a star's proper motion is equal to the sum of the squares of the star's proper motion in parallel and in N.P.D., we have only to add the respective sums of squares in the two tables given above, to deduce the following table:—

Sums of Squares of full Proper Motion.

		Uncorrected.	Corrected.	N.
Division	1	8·7868	6·9006	9
	2	2·4094	2·7097	55
	3	14·2633	14·3178	146
	4	13·4122	11·9863	238
	5	82·8461	81·1528	330
	6	19·4308	18·6126	368
	7	0·8765	0·8119	21
	Sum	142·0251	136·4917	1167

Let us first apply formulæ (2) and (6), (remembering that formula (2) is true for the whole celestial sphere). We have from them,—

$$\text{Uncorrected sum of squares} = \frac{2\,N}{3}\,(p^2 + P^2),$$

$$\text{Corrected sum of squares} = \frac{2\,N\,.\,p^2}{3}.$$

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Hence

$$P^2 = p^2 \left(\frac{\text{correction}}{\text{corrected sum}}\right).$$

Applying this successively to the several divisions, we obtain

For Division 1	$P = p$ (·52)
2	P has an imaginary value
3	P has an imaginary value
4	$P = p$ (·34)
5	$P = p$ (·18)
6	$P = p$ (·21)
7	$P = p$ (·28)

The results for the second and third divisions suffice to show that the numbers of stars which fall under these heads are insufficient for a satisfactory determination; and therefore, *à fortiori*, the number of stars in division 1 is insufficient. Hence the result $P = p$ (·52) must be dismissed as valueless. But even at this stage of the inquiry we begin to recognise that there must be something wrong about our assumptions. For even if the numbers 55 and 146 were not in themselves large enough to lead us to expect a satisfactory evaluation for stars in the second and third divisions, yet the fact that the Sun's motion in space has been correctly deduced from a smaller number of stars would justify such an expectation.

I should be led then to suspect from this evidence, that if divisions 1, 2, and 3, had been taken together, a more satisfactory conclusion would have been arrived at, notwithstanding the apparent necessity of assigning different distances to stars in these divisions. However, it is not possible to determine how far this suspicion is justified without going over Mr. Dunkin's labours afresh, with changed assumptions; and I have no leisure for the long processes of calculation this would involve.

Next we may notice that the results for divisions 4, 5, 6, and 7, are very far from satisfactory. I cannot think it credible that the real solution of the difficulty involved in the smallness of the observed corrections is to be found in the assumption that the mean motion of stars of the fourth magnitude is three times as great as the Sun's, the mean motion of fifth-magnitude stars nearly six times as great as the Sun's, and so on.

If we try the effect of diminishing the assumed distances of stars of the 4th, 5th, and 6th divisions, so as to accord with the observed relations, on the assumption that in reality $P = p$, we have (the assumed distances being 3·76, 5·44, and 7·86,* respectively) the following results:—

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* Struve's values.

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For Division 4 Distance = $3{\cdot}76 \times {\cdot}34 = 1{\cdot}28$
5 ,, = $5{\cdot}44 \times {\cdot}18 = 0{\cdot}98$
6 ,, = $7{\cdot}86 \times {\cdot}21 = 1{\cdot}65$

As division 7 includes but 21 stars, we cannot expect any trustworthy results from treating it in the same way.

The evidence thus far seems strongly opposed to accepted views respecting the distances of stars of the smaller visible magnitudes. A further inquiry on this point seems, therefore, suggested. And it is obvious that in the above table of sums of squares we have the means of testing how far the assumed distances of the stars of various orders accord with the observed proper motions. I make the assumption that the Sun's motion is equal to the average proper motion of the fixed stars. This assumption affects the actual, but not the relative, values of the distances which result from the following processes.

We have then formula (7) to deal with. Applying it to the successive orders of stars, *i.e.* putting $\frac{4\Sigma}{3}\, p^2$ successively equal to the values tabulated in the foregoing columns of uncorrected sums of squares, we obtain,

	Apparent Proper Motion.	Resulting Distance.
Division 1	″ 0·857	1
2	0·182	4·7
3	0·268	3·2
4	0·208	4·1
5	0·433	2·0
6	0·191	4·5
7	0·173	5·0

This result is unsatisfactory in the extreme. We find stars of the second magnitude placed (according to this mode of estimating their distances) further from us than stars of the 3rd, 4th, 5th, and 6th magnitudes.

Remembering the evidence we have already had, that (1) there is something erroneous in our assumptions respecting star-distances; and secondly, that small numbers of stars are insufficient for our guidance, let us apply a test which there ought to be no mistaking. Let us divide the stars into two sets, the first including divisions 1, 2, 3; the second, the remaining divisions and let us apply formula (7) to these sets.

We obtain,

For Set 1, Apparent Proper Motion,

$$= \sqrt{\frac{3}{4}\left(\frac{25{\cdot}4595}{210}\right)} = 0''{\cdot}3015,$$

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For Set 2, Apparent Proper Motion

$$= \sqrt{\frac{3}{4}\left(\frac{116{\cdot}5656}{957}\right)} = 0''{\cdot}3022.$$

This result would make the mean distance of stars of the first three magnitudes equal to (or *very slightly less* than) the mean distance of stars of the next three magnitudes!

I am very far from supposing that this result accurately represents the relations subsisting among the stars; but I do think that it suffices to render the usually accepted views respecting stellar distribution wholly untenable. Remembering that whatever theory we form regarding the relation between the apparent brilliancy and the real distance of the stars, we must yet recognise the fact that the stars are at very various distances from us. I think it must be admitted that the apparent brightness of a star is, to a certain extent, an argument of relative proximity. A large proper motion is also an argument of relative proximity. If the two indications agreed either for separate stars or for sets of stars, arranged according to apparent brilliancy, there would be no difficulty. As we find, however, that there is no such agreement, we are forced to consider whether brightness or large apparent motion is the stronger evidence of proximity. Judging from the analogy of the solar system, in which the range in the variations of magnitude is enormously greater than the range in the variations of velocity, we seem strongly led to look on the proper motions of stars as in reality the best evidence we have respecting their distances. But this conclusion is very much strengthened when we remember that the dynamical conditions in the sidereal system must be much more unfavourable to the occurrence of wide variations of velocity, than the conditions which prevail in a system of bodies circling around a central body enormously large compared with any of its dependent orbs.

I think, then, that I may fairly look upon the above inquiry as affording very striking evidence in favour of the view I had formed from other considerations, that the assumed estimate of the distances of the smaller stars has been greatly overrated. And as this conclusion may obviously be extended to yet smaller stars, I think that I have not been deceived in looking upon the relations which subsist between the Milky Way and the lucid stars in its neighbourhood as very much more intimate than has been commonly supposed. I believe that future researches will prove, not only that the Milky Way as a whole is much nearer than we have been imagining, but that portions of it are absolutely nearer to us than the brightest of the single stars. That parts of the Milky Way, for instance, in the neighbourhood of *α Centauri* (the nearest of the fixed stars, so far as is yet known), are nearer to us than that star, I think the whole aspect of the galaxy in that neighbourhood suffices to suggest, if not to demonstrate.